**1.5**

**Remainder Theorem Supplementary Notes**

**(Supplement to pgs. 9, 45-49)**

Long Division –

**Division Algorithm for Polynomials :**

Let *f*(*x*) and *d*(*x*) be polynomials with the degree of *f* greater than or equal to the degree of *d*, and *d*(*x*) 0. Then there are unique polynomials *q(x)* and *r(x),* called the **quotient** and **remainder**, such that

*f(x) = d(x)• q(x) + r(x)*  (where *d(x)* is the **divisor**)

where either *r(x)* = 0 or the degree of *r* is less than degree of *d*.

If  *r(x)*= 0 then *d(x)* divides evenly into *f(x).*

Example:

Use long division to find the quotient and remainder when is divided by  Write a summary statement in both polynomial and fraction form.

Solution:



The division algorithm yields the polynomial form

.

This can also be written as

 (Fraction form)

**Remainder theorem:** If a polynomial *f(x)* is divided by *x – k,* then the remainder *r = f(k).*

Example:

Find the remainder when  ** is divided by x – 2.



**Factor Theorem:** A polynomial function *f(x)* has a factor *x - k* if and only if *f(k)* = 0.

**Example using the Remainder Theorem:**

Find the remainder when is divided by

1. x – 2 b) x + 1 c) x + 4

**Solution:**

1. We can find the remainder without doing long division! Using the Remainder Theorem with *k* = 2 we find that

.

1. 
2. 

Because the remainder in part c) is 0, x + 4 divides evenly into . So, it is a factor of , and -4 is a solution of . -4 is also an x-intercept of the graph of .

**Synthetic division**

**Example:**

Divide using synthetic division and write a summary statement in fraction form.

**Solution:**

The zero of the divisor x – 3 is 3, which we put in the divisor position. Because the dividend is in standard form, we write its coefficients in order in the dividend position, *making sure to use zero as a placeholder for any missing term.* We leave space for the line for products and draw a horizontal line below the space. (See below.)

* Because the leading coefficient of the dividend must be the leading coefficient of the quotient, copy the 2 into the first quotient position.



* Multiply the zero of the divisor (3) by the most recently determined coefficient of the quotient (2). Write the product above the line and one column to the right.
* Add the next coefficient of the dividend to the product just found and record the sum below the line in the same column.
* Repeat the “multiply” and “add” steps until the last row is completed.



The last line of numbers are the coefficients of the quotient polynomial and the remainder. The quotient must be a quadratic function. (Why?) So the quotient is and the remainder is 0. So we conclude that .